Journal of Engineering Physics and Thermophysics, Vol. 66, No. 3, 1994

NUMERICAL SIMULATION OF GASDYNAMIC PROCESSES IN THE CASE OF CATASTROPHIC VOLCANIC ERUPTIONS

I. M. Kozlov, G. S. Romanov, and A. E. Suvorov

UDC 519.6

Numerical simulation of the flow of air originating as a result of volcanic eruption is carried out with the parameters corresponding to those figuring in the eruption of the Krakatau volcano in 1883 (with an energy of 100-1000 Mton of TNT equivalent). The results of the calculation are compared with the consequences of the disaster known from the literature.

Volcanic eruptions having the character of disasters occur very rarely. Thus, an eruption of the Tambora volcano type (1815, the energy is estimated to be equal to 25 Gton of TNT equivalent [1]), which is the largest of those mentioned in history, has a repetition frequency of one in 10 thousand years, and an eruption of the Krakatau volcano type (1883, the energy by various estimates amounted to 100-1000 Mton of TNT equivalent) recurs approximately once in 100 years [2]. However, their consequences are so enormous that it is of definite interest to try to reproduce the observed parameters and the dynamics of the development of such eruptions by means of direct numerical simulation of the phenomenon on the basis of the equations of gas dynamics.

This problem is considered in the present work on the basis of an extremely simplified model of the phenomenon: in an axisymmetric geometry at the initial instant of time the entire energy of the explosion consists in the thermal energy of a gas having the initial temperature T and the pressure p and occupying a hemispherical volume of radius R located on the surface of the earth. According to estimates given in [3], for the eruption of the Krakatau volcano $T = 1400^{\circ}$C, $p = 420$ atm, and $R = 4.8$ km. An estimation of the total energy E enclosed in the hemisphere on the assumption that the gas contained in it has thermodynamic characteristics close to the characteristics of air [4] gives $E = 374$ Mton. According to literature data [3], the volcanic ashes ascended 18 miles and the dust ascended 50 miles.

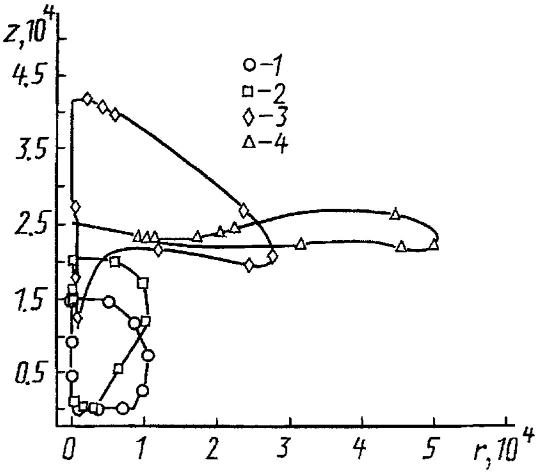
The calculation is performed by the method of large particles in a cylindrical system of coordinates [5]. Let us dwell in more detail on the method used to calculate the equilibrium atmosphere. At the initial instant, when the explosion-induced perturbation is absent, the parameters of the atmosphere satisfy the equation of hydrostatic equilibrium [6]

$$\frac{\partial p}{\partial z} = -\rho g. \tag{1}$$

However, the solution of Eq. (1) is unsatisfactory because it is not an exact solution of the numerical equations of motion. Therefore, nonphysical motion begins in the unperturbed atmosphere. This may quickly lead to the development of instability when this motion is directed toward the inside of the computational domain. From this viewpoint, the parameters of the equilibrium atmosphere should be found from the hydrostatic equation that results from the numerical equations. For the method of large particles this equation has the form

$$p_{i+1/2} - p_{i-1/2} + a_i p_i = 0, \tag{2}$$

Academic Scientific Complex "A. V. Luikov Heat and Mass Transfer Institute of the Academy of Sciences of Belarus," Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 66, No. 3, pp. 263-266, March, 1994. Original article submitted July 15, 1992.



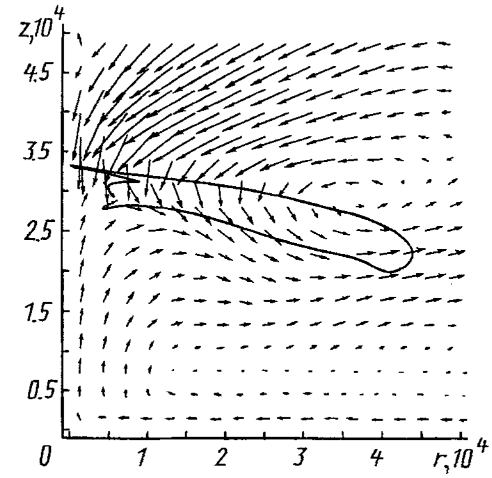


Fig. 1. Transformation of a cloud of the products of volcanic activity: (1) t = 30 sec; (2) 90; (3) 210; (4) 450 sec. z, r, mm.

Fig. 2. Field of velocities at t = 360 sec. The solid line shows the boundary of the cloud of the products of volcanic activity. V_{max} = 0.28 km/sec.

where $a_i = g\Delta z_i \rho_i/p_i$, the integral indices pertain to the middles of the cells, and the half-integral ones to their boundaries. Since the pressures on the boundaries are obtained by means of interpolation over two adjacent cells, Eq. (2) is a three-point equation and, in contrast to Eq. (1), requires two boundary conditions. One boundary condition is the parameters of the atmosphere on the surface of the earth, whereas the other cannot be formulated on physical grounds. Thus, the solution of Eq. (2) involves a free parameter, which should be selected in such a way that it could best fit the solution of Eq. (1). This problem is solved most simply for the case of an isothermal atmosphere, an ideal gas, and cells constant in size, when Eq. (2) can be solved analytically:

$$p_i = p_0 \left[\pm \sqrt{1 + (g\,\Delta z/c^2)^2} - g\,\Delta z/c^2 \right]^i . \tag{3}$$

Of the two solutions, one (with the positive root) approximates the solution of Eq. (1) well, whereas the other alternates in sign, with its absolute value increasing with height. It is evident that the alternating-sign solution should be discarded. In the general case, one cannot carry out such an analysis, and from the family of solutions of Eq. (2) we select the smoothest one, thereby discarding the alternating-sign part of the solution. The algorithm of the solution is the following:

$$\beta_{i-1} = \frac{\gamma_{i-1}}{(1 + \gamma_{i-1})(\alpha_i + a_i) - 1}, \quad i = n, \ldots, 1, \tag{4}$$

$$p_{i+1} = \beta_i p_i, \quad i = 0, \ldots, n, \tag{5}$$

$$\gamma_i = \Delta z_{i+1}/\Delta z_i, \quad \alpha_i = (\gamma_i + \beta_i)/(1 + \gamma_i).$$

Arbitrarily assigning β_n, which is precisely the free parameter, we solve Eq. (4). After that, with the aid of the boundary condition at the surface of the earth we solve Eq. (5). The criterion for the smoothness of the solution is the minimum of the function

$$F(\beta_n) = \Sigma \ln^2 \frac{\beta_{i+1}\beta_{i-1}}{\beta_i^2}. \tag{6}$$

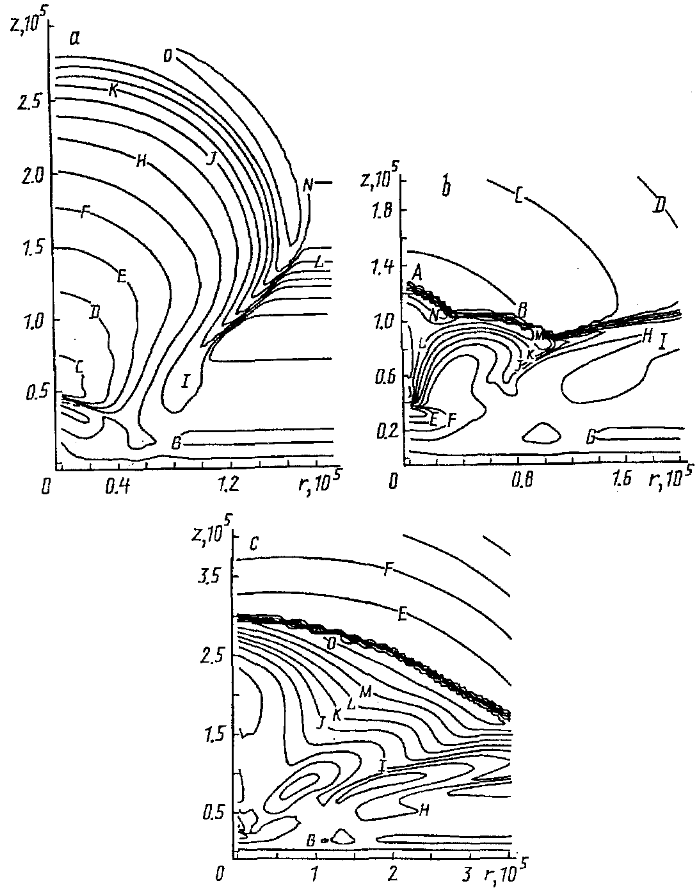


Fig. 3. Temperature field ((a) t =180, (b) 330, (c) 450 sec): A, 50 K; B, 63; C, 81; D, 104; E, 133; F, 173; G, 217; H, 278; I, 355; J, 454; K, 581; L, 742; M, 949; N, 1212; O, 1550 K.

In the case of a nonideal gas Eqs. (4)-(6) are solved with the help of iterations, with the values of a_i being taken from the previous iteration. To attain the computer accuracy, no more than 5-6 iterations are required. In the case where solution (3) is applicable, the minimum of functional (6) gives the same solution. Empirical checking showed that this approach always gives a good equilibrium atmosphere. Assuming that the dependence of temperature on height is determined by the standard atmosphere model [7] and using the equation of state of air [4], we obtain the equilibrium distribution of the parameters of the atmosphere. It satisfies the numerical equations of motion exactly.

Let us consider some results of calculations carried out up to the instant of time t = 7.5 min from the beginning of eruption. The computational grid had 120 cells along the height and 80 cells over the radius. At the beginning of the calculation this grid covered a region 9.6 km in height and of radius 6.4 km. In the course of the calculation, when the motion that arose reached the boundaries of the computational domain, these boundaries were expanded, so that at the end of the computation the grid covered a region 600 km in height with a radius of 350 km.

To visualize the motion of volcanic products, at the beginning of the calculation they were surrounded by markers, which thereafter were transported with the gas flow. The transformation of this cloud with time is shown in Fig. 1.

Let us consider parameters that can be compared with familiar descriptions of this eruption. At t = 30 sec a marked rise of the cloud in the atmosphere begins. At this time the shock wave radius is equal to about 3 km. At t = 90 sec the shock wave height is equal to about 90 km, whereas in the horizontal direction a distance of about 50 km is covered. The pressure at the wave front on the surface of the earth is about 1.2 atm. At t = 210 sec the shock wave rises to 400 km and reaches a distance of more than 80 km along the earth, with the pressure here amounting to about 1.15 atm. The dust cloud has a height of 20 km at $t = 90$ sec and nearly 40 km at $t = 210$ sec. When $t = 300$ sec, the shock wave rises to about 600 km, the path covered along the surface of the earth is about 120 km, the cloud rises to about 40 km, and the excess pressure at the front is somewhat less than 0.1 atm. It should be noted that according to observations [3], at a distance of 100 miles glass was broken, which requires, as is known [8], an excess pressure of about 0.07 atm, i.e., the calculation is well confirmed by the observations, which favors the selected value for the energy of the eruption. At $t = 200$ sec at a height of about 100 km intense downstream motion of air masses sent upward by the eruption begins. At $t = 450$ sec this leads to a lowering of the dust cloud to a height of about 30 km. This agrees satisfactorily with the observed height of 18 miles. The velocity field in the region of the cloud at $t = 360$ sec is shown in Fig. 2.

At the initial stage of the eruption enormous masses of near-earth dense air are ejected to a considerable height where they expand and cool drastically (see Fig. 3a). Thereafter, under the action of gravity this air begins to fall downstream, developing velocities of the order of a kilometer per second and impinging on relatively immobile denser layers of air. This leads to the formation of an intense shock wave, which, propagating upward, warms the cooled layers of air (Figs. 3b, c). This motion is repeated several times with a gradually decreasing amplitude.

The results given above verify that direct numerical simulation of global catastrophic phenomena is possible.

NOTATION

t, time; T, temperature; p, pressure; r, current radius; E, explosion energy; z, current height; ρ, density; g, free fall acceleration; c speed of sound; n, number of cells for (1); Δz, size of a cells along the height; $F(\beta)$, smoothness criterion; α, a, β, γ, coefficients in various equations.

REFERENCES

1. Climatic and Biological Consequences of Nuclear War [in Russian], Moscow (1978).
2. A. Kondratov, The Deluge. Myths and Reality [in Russian], Leningrad (1984).
3. A. G. Galapopulos and E. Bacon, Atlantis. Truth behind a Legend [Russian translation], Moscow (1983).
4. N. M. Kuznetsov, Thermodynamic Functions and Shock Adiabats of Air at High Temperatures [in Russian], Moscow (1965).
5. O. M. Belotserkovskii and Yu. M. Davydov, The Method of Large Particles in Gas Dynamics [in Russian], Moscow (1982).
6. L. D. Landau and E. M. Lifshits, Hydrodynamics [in Russian], Moscow (1986).
7. Handbook on Geophysics [in Russian], Moscow (1965).
8. Ya. B. Zel'dovich and B. E. Gel'fand (eds.), Explosion Phenomena. Estimation and Consequences [in Russian], Moscow (1986).